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Maths assignment

Que-1 find the domain of the function.

$$f(x) = \frac{x^2-4}{x-2}$$

→ Let $y = \frac{x^2-4}{x-2}$

→ $y = x+2$ when $x \neq 2$.

Now $\frac{x^2-4}{x-2}$ is not defined only when $x=2$, therefore

Domain of $y = \frac{x^2-4}{x-2}$ is $R-\{2\}$.

e-2 find the range of the function $f(x) = \sin^2 x + \cos^2 x$.

-2. The function $f(x) = \sin^2(x) + \cos^2(x)$ always evaluates to 1 for any value of x. This is because of the trignometric identity: $\sin^2(x) + \cos^2(x) = 1$.

So, the range of the function is $[1,1]$, which means that the function $f(x)$ only takes on the value of 1 for all real value of x.

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Que-3 Check continuity of $f(x) = x \cdot \sin(1/x)$ at $x=0$

Solve $f(x) = x \sin\left(\frac{1}{x}\right)$ for $x=0$ and $f(x)=0$ for $x=0$

for a continuous function $\underset{x\to a}{L.H.L} = \underset{x\to a}{R.H.L} = f(a)$

Let LHL.

$\Rightarrow \lim_{x\to 0^-} x \sin\frac{1}{x}$

$\Rightarrow \lim_{n\to 0} (0-n) \sin\left(\frac{1}{-n}\right).$

$\Rightarrow \lim_{n\to 0} -n\sin\left(\frac{-1}{n}\right)$

$\Rightarrow \lim_{n\to 0} n \sin\frac{1}{n}$ $[\because \sin(-\theta) = -\sin\theta].$

$\Rightarrow 0 \sin(\infty).$

$\Rightarrow 0$

$\Rightarrow$ R.H.L.

$\Rightarrow \lim_{n\to 0^+} x \sin\frac{1}{x}$

$\Rightarrow \lim_{n\to 0} (0+n) \sin\frac{1}{0+n}$

$\Rightarrow \lim_{n\to 0} n\sin\frac{1}{n} = 0 \cdot \sin\infty$

$\Rightarrow 0$

$\Rightarrow$ So, $L.H.L = R.H.L = f(0).$

$\Rightarrow$ So function is continuous.

Que-4 Verify Rolle's theorem for

$$f(x) = \begin{cases} x^2+1 ; & 0 \le x \le 1 \\ 3-x ; & 1 \le x \le 2. \end{cases}$$

→ $f(x)$ is defined in $[0,2]$.
Also $f(x) = x^2+1$ in $[0,1]$.
is continuous. & differentiable in $[0,1$

Similarly
$f(x) = 3-x$ in $[1,2]$ is also conti
as well as differiable $[0,1]$

→ At $x=1$, continuty at $x=1$

$$L.H.L = \lim_{x \to 1^-} f(x)$$

$$= \lim_{x \to 1} (x^2+1) = 2.$$

$$R.H.L = \lim_{x \to 1^+} f(x) = \lim_{x \to 1} 3-x = 3-1 = 2.$$

Also $\lim_{x \to 1} f(x) = 2$

$\because$ L.H.L = R.HL = $f(x) = 2$

Hence it is continuous.

Differentiability at $x=1$

$$R.H.S = \lim_{x \to 0} \frac{f(1+h) - f(1)}{h}$$

$$= \lim_{h \to 0} \frac{[3-(1+h)] - 2}{h}$$

$$= \lim_{h \to 0} \frac{2-h-2}{h} = \lim_{h \to 0} -\frac{h}{h} = -1$$

$$L.H.D = \lim_{h \to 0} \frac{f(1-h) - f(1)}{-h} = \lim_{h \to 0} \frac{[(1-h)^2 + 1] - 2}{-h}$$

$$= \lim_{h \to 0} \frac{\cancel{1} + h^2 - 2h + \cancel{1} - \cancel{2}}{h} = \lim_{h \to 0} \frac{h(h-2)}{h}$$

$$= \lim_{h \to 0} -(h-2)$$

$$\Rightarrow 2$$

R.H.D $\neq$ LH.D

This is not differentiable.

Que-5 If $f(x)$ is continous in $[a,b]$ and diff. in (a,b), The $\forall x \in (a,b)$

(a) $f'(x)=0 \Rightarrow f(x)$ is a constant function.

(b) $f'(x)>0 \Rightarrow f(x)$ is Monotone increasing $f(x)$?.

(c) $f'(x)<0 \Rightarrow f(x)$ is monotone decreasing $f(x)$?

→ Proof :→ suppose $x_1 < x_2$ are two points in $[a,b]$ s.t. $a<x_1<x_2<b$

$[x_1, x_2] \subset [a,b]$

$\because$ $f(x)$ is continous in $[x_1, x_2]$

& $f(x)$ is diff. in (x_1, x_2).

By the LMVT, $\exists$ c (x_1, x_2) s.t.

$$f'(c) = \frac{f(x_2)-f(x_1)}{x_2-x_1}$$

$\forall x \in (a,b)$

(a) $f'(x)=0 \Rightarrow f'(c)=0 = \dfrac{f(x_2)-f(x_1)}{x_2-x_1}$

$\Rightarrow f(x_2)-f(x_1)=0$

$\Rightarrow f(x_2) = f(x_1)$

$f(x)$ is a constant $f x^n$ in (a,b)

(b) $f'(x)>0 \Rightarrow f'(c)>0 \Rightarrow \dfrac{f(x_2)-f(4)}{x_2-x_1} > 0$

$\Rightarrow$ $f(x_2)-f(x_1)>0$

$\Rightarrow$ $f(x_2)>f(x_1)$

$\Rightarrow$ $f(x)$ is monotone increasing in (a,b)

) $f'(x) < 0 \Rightarrow f'(c) < 0$

$$\Rightarrow \frac{f(x_2) - f(x_1)}{x_2 - x_1} < 0$$

$$\Rightarrow f(x_2) - f(x_1) < 0$$

$$\Rightarrow f(x_2) < f(x_1)$$

$f(x)$ is Monotone decreasing in (a, b)